# EE 330 Lecture 23

Small Signal Analysis

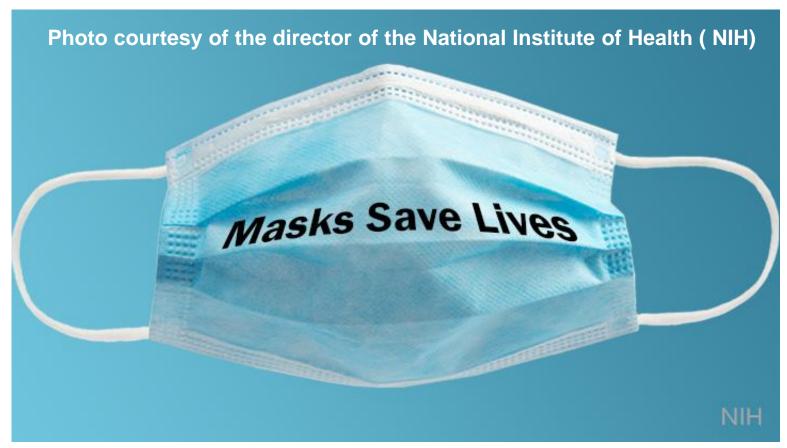
## Exam Schedule

Exam 1 Friday Sept 24

Exam 2 Friday Oct 22

Exam 3 Friday Nov 19

Final Tues Dec 14 12:00 p.m.



As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

# Amplification with Transistors

From Wikipedia: (Oct. 2019)

An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that can increase the <u>power</u> of a <u>signal</u> (a timevarying <u>voltage</u> or <u>current</u>).

What is the "power" of a signal?

Can an amplifier make decisions?

Does Wikipedia have such a basic concept right?

## Operating Point of Electronic Circuits

Often interested in circuits where a small signal input is to be amplified (e.g. V<sub>M</sub> in previous slide is small)

The electrical port variables where the small signals goes to 0 are termed the Operating Points, the Bias Points, the Quiescent Points, or simply the Q-Points

By setting the small signal inputs to 0, it means replacing small voltage inputs with short circuits and small current inputs with open circuits

When analyzing small-signal amplifiers, it is necessary to obtain the Q-point

When designing small-signal amplifiers, establishing of the desired Q-point is termed "biasing"

- Capacitors become open circuits (and inductors short circuits) when determining Q-points
- Simplified dc models of the MOSFET (saturation region) or BJT (forward active region) are usually adequate for determining the Q-point in practical amplifier circuits
- DC voltage and current sources remain when determining Q-points
- Small-signal voltage and current sources are set to 0 when determining Q-points

What is a dependent source?

Did you ever see one in the EE 201 Course?

Did you have them in your parts kit for EE 201?

Did you ever see one in the EE 201 Laboratory?

Did you ever see one in the EE 230 Laboratory?

Can you buy one from Digi Key?

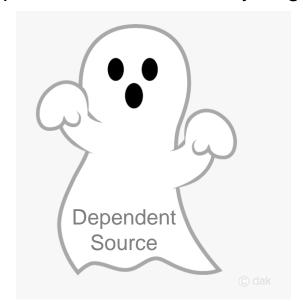


World's Largest Selection of Electronic Components Available for Immediate Shipment!®

Digi-Key has over 9 million different parts from over 1000 suppliers!

## What is a dependent source?

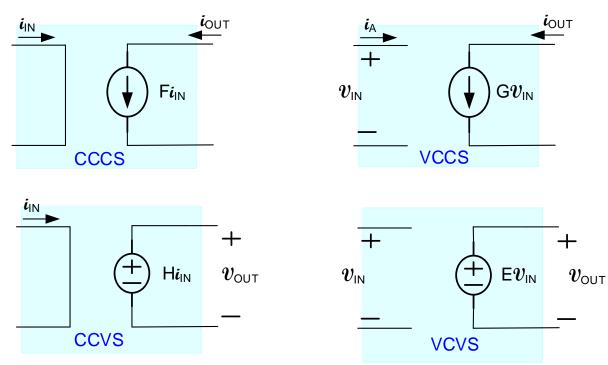
Will you suddenly find dependent sources after you graduate?



Do dependent sources really exist?

Why do we place so much emphasis on dependent sources in EE 201?

## What is a dependent source?

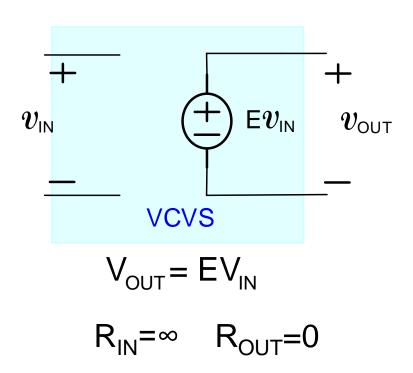


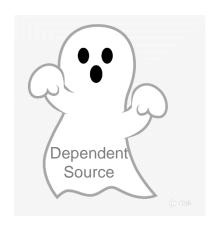
The four basic dependent sources!

Two-port networks with infinite or zero input and output impedances



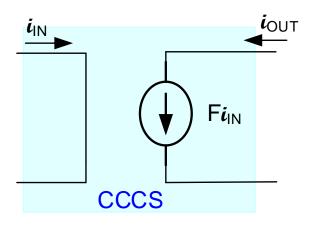
Observe, as an example,





Does this have anything in common with a Voltage Amplifier?

Observe, as an example,

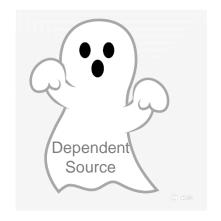


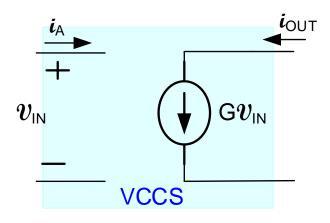


$$I_{OUT} = FI_{IN}$$
 $R_{IN} = 0$   $R_{OUT} = \infty$ 

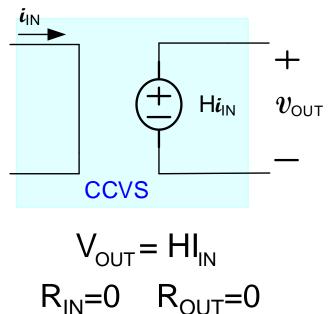
Does this have anything in common with a Current Amplifier?

Observe, as an example,





$$I_{OUT} = GV_{IN}$$
 $R_{IN} = \infty$   $R_{OUT} = \infty$ 



What about these dependent sources?

# **Amplification with Transistors**



Often the voltage gain of an amplifier is larger than 1

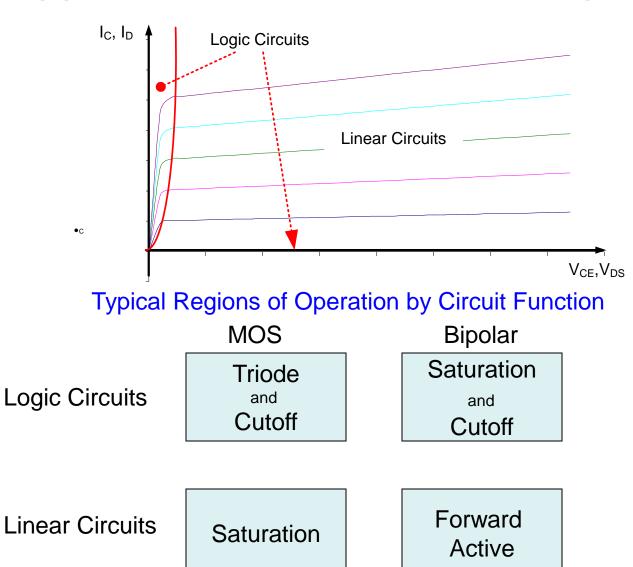
$$V_{OUT} = A_{V}V_{IN}$$

Often (but not always) the power dissipated by  $R_L$  is larger than the power supplied by  $V_{IN}$ 

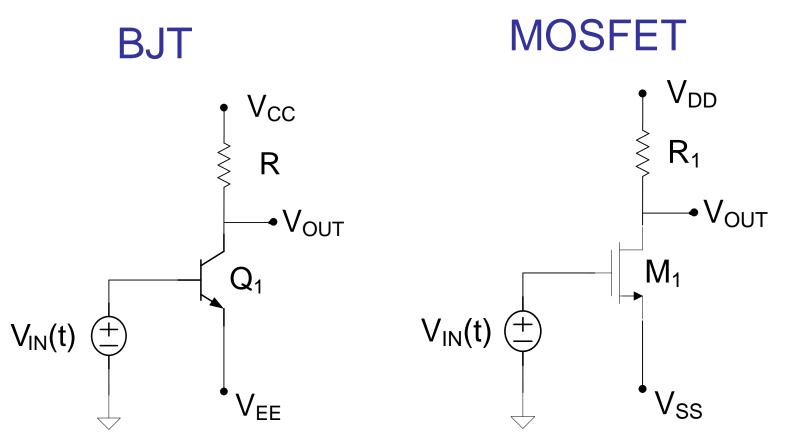
An amplifier can be thought of as a dependent source that was discussed in EE 201

Input and output variables can be either V or I or mixed

## Applications of Devices as Amplifiers



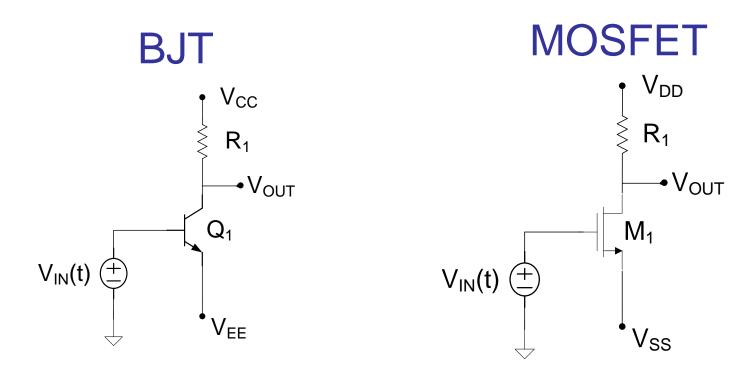
## Consider the following MOSFET and BJT Circuits



Assume BJT operating in FA region, MOSFET operating in Saturation Assume same quiescent output voltage and same resistor R<sub>1</sub> Note architecture is same for BJT and MOSFET circuits

One of the most widely used amplifier architectures

## Consider the following MOSFET and BJT Circuits



- MOS and BJT Architectures often Identical
- Circuits are Highly Nonlinear
- Nonlinear Analysis Methods Must be used to analyze these and almost any other nonlinear circuit

# Methods of Analysis of Nonlinear Circuits

KCL and KVL apply to both linear and nonlinear circuits

Superposition, voltage divider and current divider equations, Thevenin and Norton equivalence apply only to linear circuits!

Some other analysis techniques that have been developed may apply only to linear circuits as well

## Methods of Analysis of Nonlinear Circuits

Will consider <u>three</u> different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

Interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course

#### 1. Nonlinear circuits with continuously differential devices

Analysis Strategy:

Use KVL and KCL for analysis

Represent nonlinear models for devices either mathematically or graphically

Solve the resultant set of nonlinear and linear equations (often differential equations) for the variables of interest

#### 2. Circuits with piecewise continuous devices

e.g. 
$$f(x) = \begin{cases} f_1(x) & x < x_1 & \text{region 1} \\ f_2(x) & x > x_1 & \text{region 2} \end{cases}$$

Analysis Strategy:

Guess region of operation

Solve resultant circuit using the previous method

Verify region of operation is valid

Repeat the previous 3 steps as often as necessary until region of operation is verified

- It helps to guess right the first time but a wrong guess will not result in an incorrect solution because a wrong guess can not be verified
- Piecewise models generally result in a simplification of the analysis of nonlinear circuits

# 3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

#### Analysis Strategy:

Use methods from previous two class of nonlinear circuits

#### More Practical Analysis Strategy:

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0)

Develop small signal (linear) model for all devices in the region of interest (around the operating point or "Q-point")

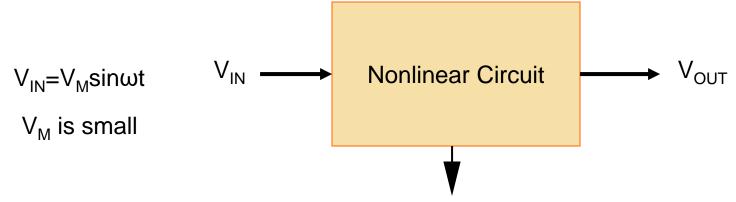
Create small signal equivalent circuit by replacing <u>all</u> devices with small-signal equivalent

#### Solve the resultant small-signal (linear) circuit

Can use KCL, DVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence

Determine boundary of region where small signal analysis is valid

## Small signal operation of nonlinear circuits



If  $V_M$  is sufficiently small, then any nonlinear circuit operating at a region where there are no abrupt nonlinearities will have a nearly sinusoidal output and the variance of the magnitude of this output with  $V_M$  will be nearly linear (could be viewed as "locally linear")

This is termed the "small signal" operation of the nonlinear circuit

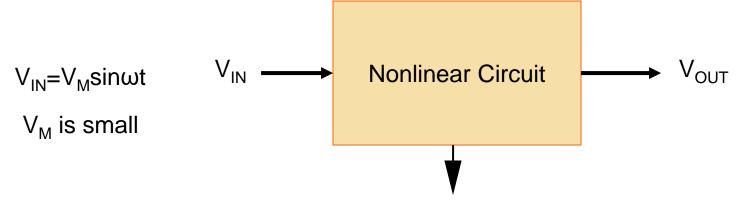
When operating with "small signals", the nonlinear circuit performs linearly with respect to these small signals thus other properties of linear networks such as superposition apply provided the sum of all superimposed signals remains sufficiently small

Other types of "small signals", e.g. square waves, triangular waves, or even arbitrary waveforms often are used as inputs as well but the performance of the nonlinear network also behaves linearly for these inputs

Many useful electronic systems require the processing of these small signals

Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

## Small signal operation of nonlinear circuits



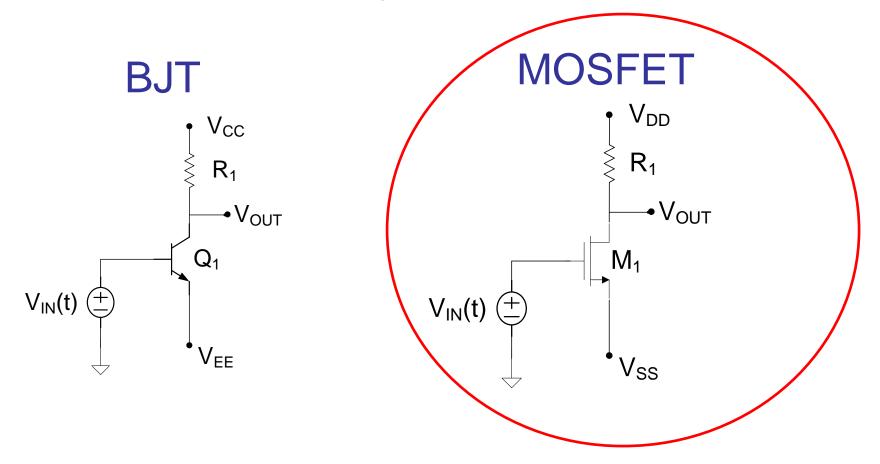
Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

#### Two key questions:

How small must the input signals be to obtain locally-linear operation of a nonlinear circuit?

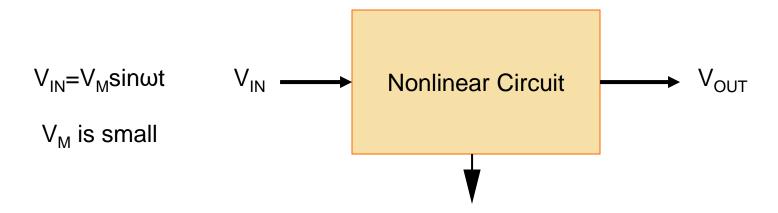
How can these locally-linear (alt small signal) circuits be analyzed and designed?

## Consider the following MOSFET and BJT Circuits

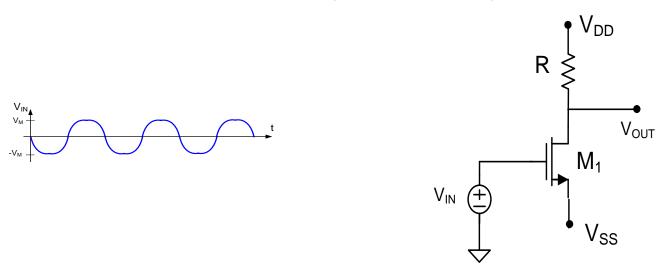


One of the most widely used amplifier architectures

## Small signal operation of nonlinear circuits

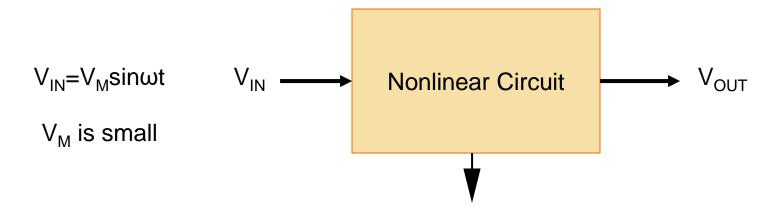


Example of circuit that is widely used in locally-linear mode of operation



Two methods of analyzing locally-linear circuits will be considered, one of these is by far the most practical

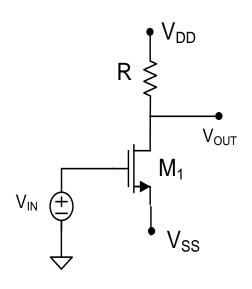
## Small signal operation of nonlinear circuits



Two methods of analyzing locally-linear circuits for small-signal excitaions will be considered, one of these is by far the most practical

- 1. Analysis using nonlinear models
- 2. Small signal analysis using locally-linearized models

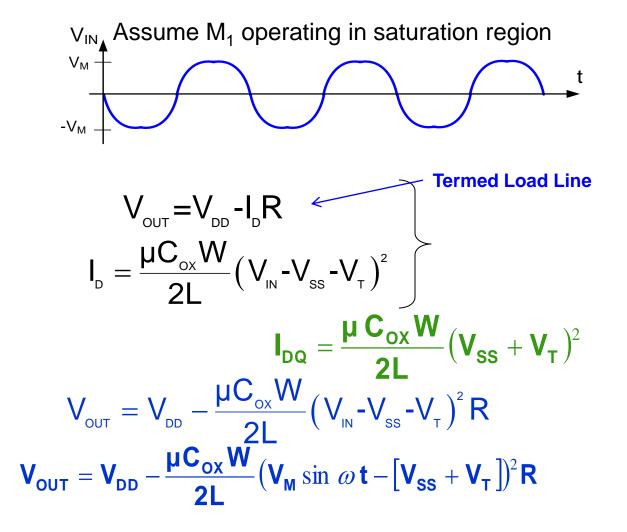
## Small signal analysis using nonlinear models

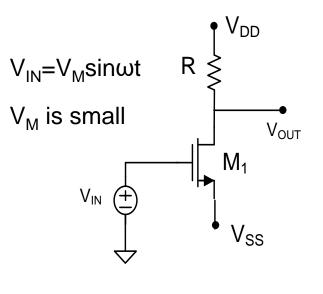


 $V_{IN}=V_{M}sin\omega t$ 

V<sub>M</sub> is small

By selecting appropriate value of  $V_{SS}$ ,  $M_1$  will operate in the saturation region





$$\mathbf{V}_{\mathsf{OUT}} = \mathbf{V}_{\mathsf{DD}} - \frac{\mu \mathbf{C}_{\mathsf{OX}} \mathbf{W}}{2 \mathsf{L}} (\mathbf{V}_{\mathsf{M}} \sin \omega \, \mathbf{t} - [\mathbf{V}_{\mathsf{SS}} + \mathbf{V}_{\mathsf{T}}])^{2} \mathsf{R}$$

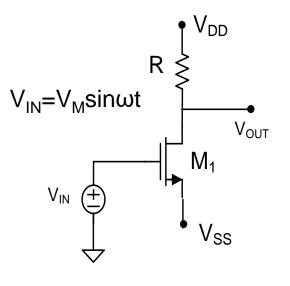
Note relationship between input and output not linear!

$$\mathbf{V}_{\text{OUT}} = \mathbf{V}_{\text{DD}} - \frac{\mu \mathbf{C}_{\text{OX}} \mathbf{W}}{2 L} \left[ \mathbf{V}_{\text{SS}} + \mathbf{V}_{\text{T}} \right]^2 \left( 1 - \frac{\mathbf{V}_{\text{M}} \sin \omega t}{\left[ \mathbf{V}_{\text{SS}} + \mathbf{V}_{\text{T}} \right]} \right)^2 \mathbf{R}$$

Recall that if x is small 
$$V_{\text{OUT}} \cong V_{\text{DD}} - \frac{\mu C_{\text{OX}} W}{2L} \left[ V_{\text{SS}} + V_{\text{T}} \right]^2 \left( 1 - \frac{2V_{\text{M}} \sin \omega t}{\left[ V_{\text{SS}} + V_{\text{T}} \right]} \right) R$$

$$V_{\text{OUT}} \cong \left\{ V_{\text{DD}} - \frac{\mu C_{\text{OX}} W}{2L} \left[ V_{\text{SS}} + V_{\text{T}} \right]^2 R \right\} + \frac{\mu C_{\text{OX}} W}{2L} \left[ V_{\text{SS}} + V_{\text{T}} \right]^2 \left( \frac{2V_{\text{M}} \sin \omega t}{\left[ V_{\text{SS}} + V_{\text{T}} \right]} \right) R$$

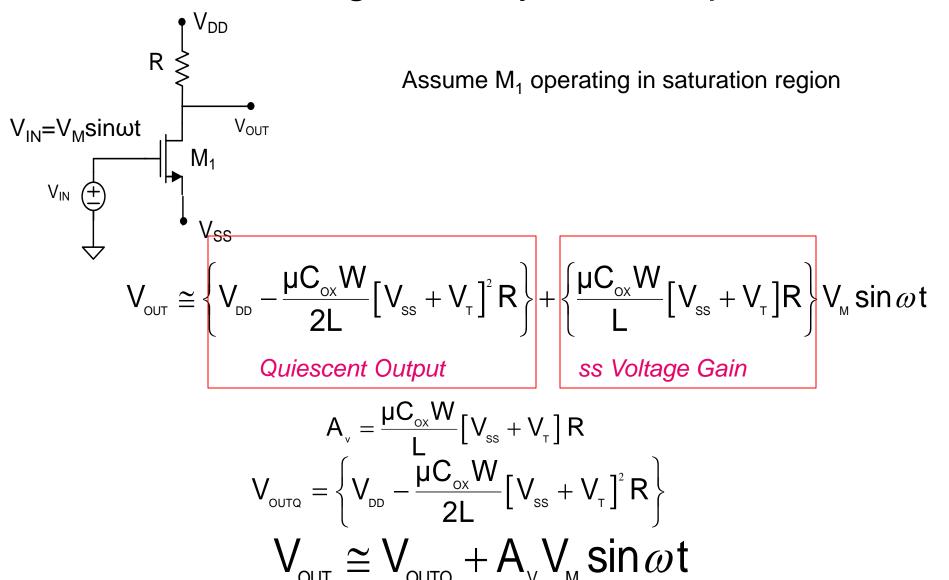
$$V_{\text{OUT}} \cong \left\{ V_{\text{DD}} - \frac{\mu C_{\text{OX}} W}{2L} \left[ V_{\text{SS}} + V_{\text{T}} \right]^2 R \right\} + \left\{ \frac{\mu C_{\text{OX}} W}{L} \left[ V_{\text{SS}} + V_{\text{T}} \right] R \right\} V_{\text{M}} \sin \omega t$$



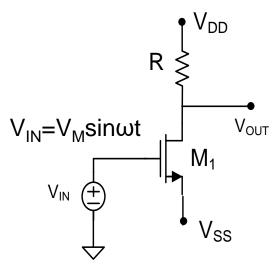
By selecting appropriate value of  $V_{SS}$ ,  $M_1$  will operate in the saturation region

Assume M<sub>1</sub> operating in saturation region

$$V_{\text{out}} \cong \left\{ V_{\text{dd}} - \frac{\mu C_{\text{ox}} W}{2L} \left[ V_{\text{ss}} + V_{\text{T}} \right]^{2} R \right\} + \left\{ \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{ss}} + V_{\text{T}} \right] R \right\} V_{\text{M}} \sin \omega t$$



Note the ss voltage gain is negative since  $V_{SS}+V_{T}<0!$ 



Assume M<sub>1</sub> operating in saturation region

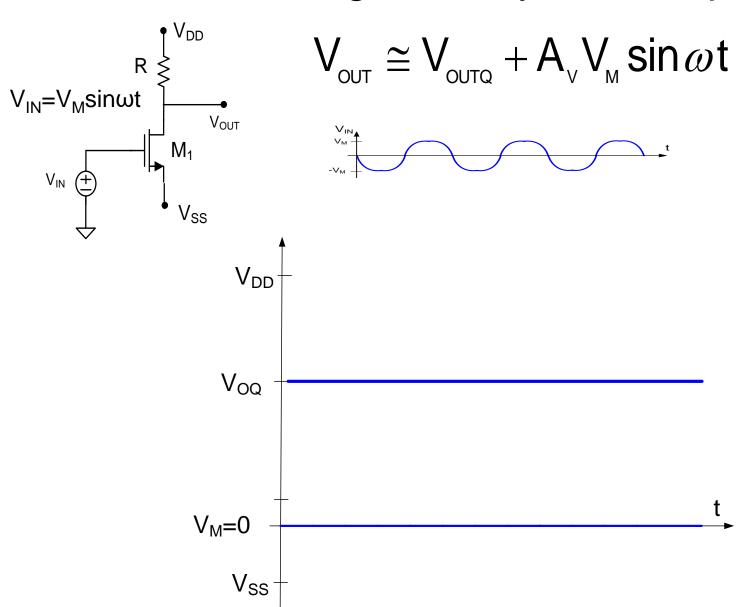
$$V_{\text{out}} \cong V_{\text{outq}} + A_{\text{\tiny V}} V_{\text{\tiny M}} \sin \omega t$$

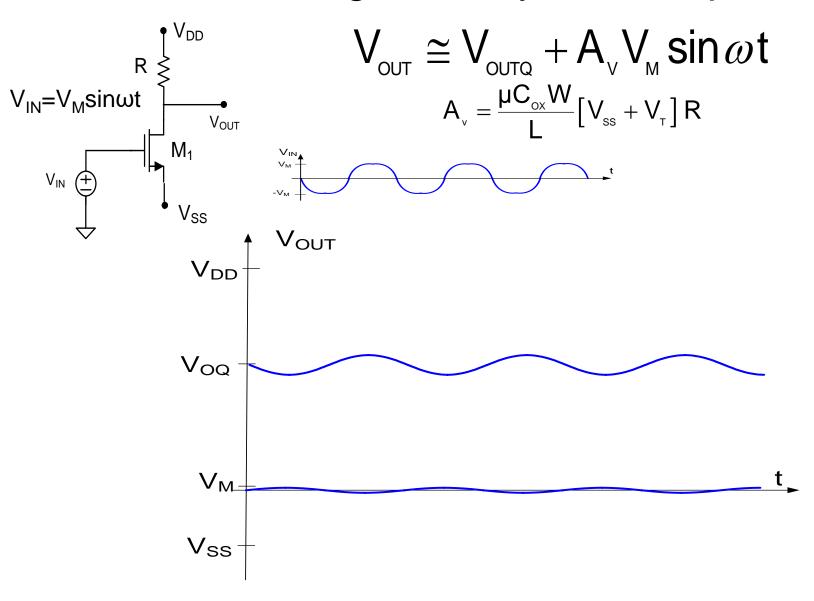
$$A_{_{\scriptscriptstyle V}} = \frac{\mu C_{_{\scriptscriptstyle OX}} W}{L} [V_{_{\scriptscriptstyle SS}} + V_{_{\scriptscriptstyle T}}] R$$

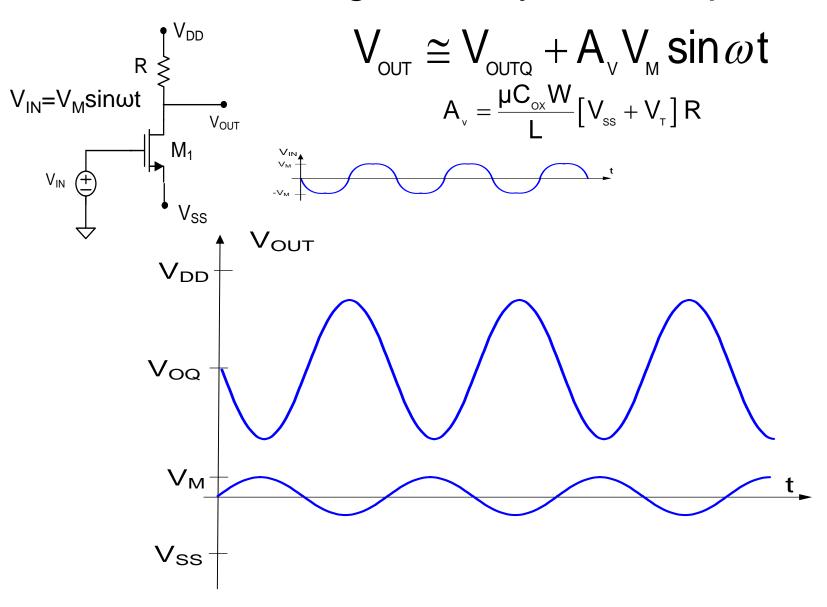
$$V_{\text{outq}} = \left\{ V_{\text{dd}} - \frac{\mu C_{\text{ox}} W}{2L} \left[ V_{\text{ss}} + V_{\text{t}} \right]^2 R \right\}$$

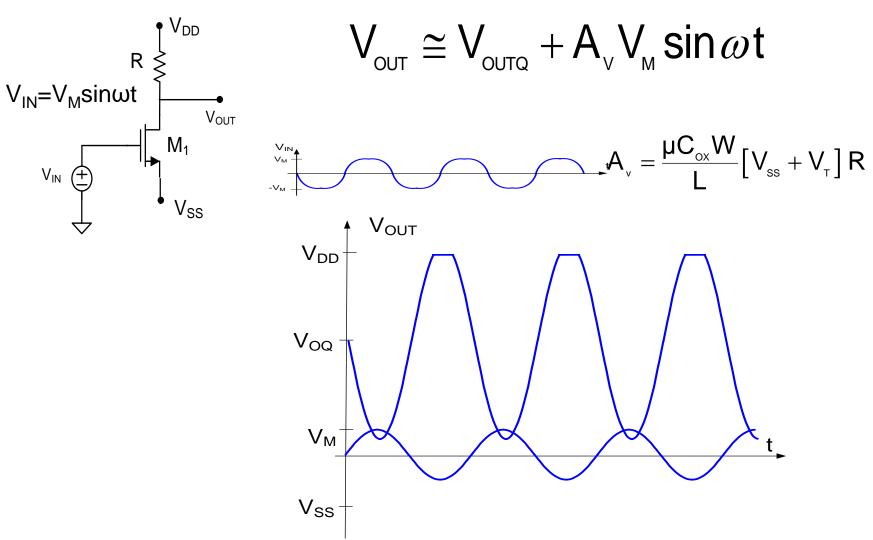
But – this expression gives little insight into how large the gain is!

And the analysis for even this very simple circuit was messy!

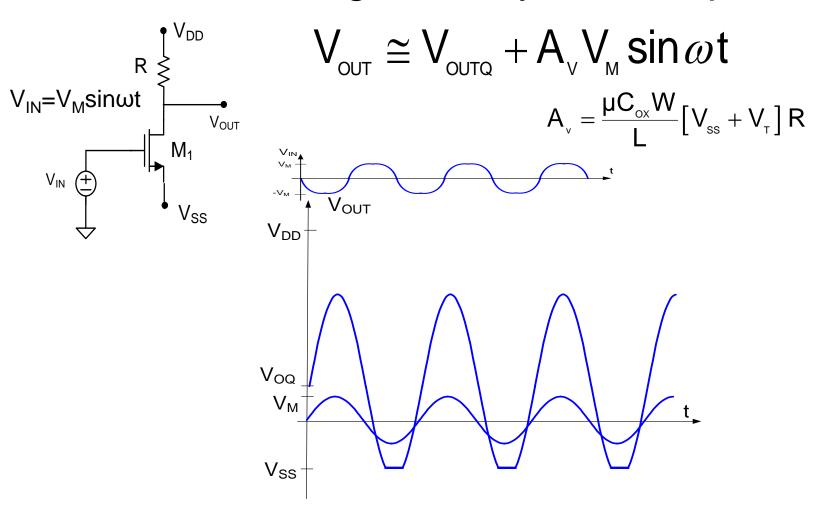




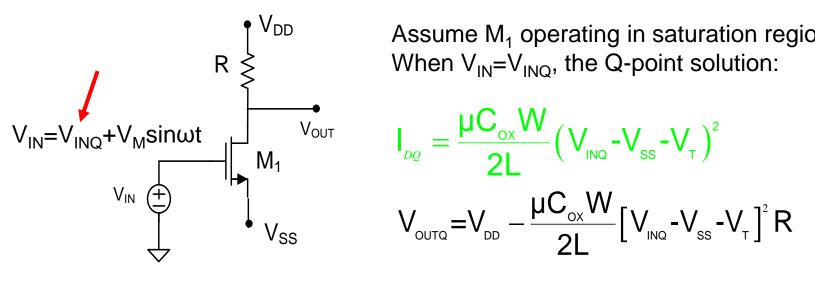




Serious Distortion occurs if signal is too large or Q-point non-optimal Here "clipping" occurs for high V<sub>OUT</sub>



Serious Distortion occurs if signal is too large or Q-point non-optimal Here "clipping" occurs for low  $V_{\text{OUT}}$ 



Assume M₁ operating in saturation region

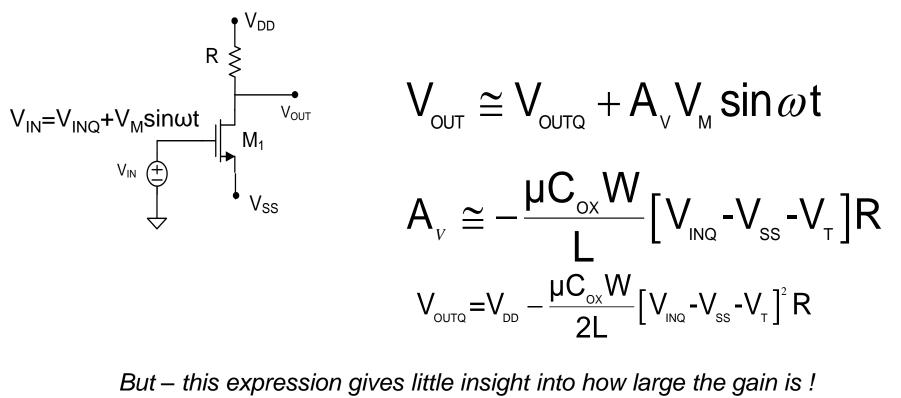
$$\begin{split} & I_{\text{DQ}} = \frac{\mu C_{\text{OX}} W}{2L} \left( V_{\text{INQ}} \text{-} V_{\text{SS}} \text{-} V_{\text{T}} \right)^2 \\ & V_{\text{OUTQ}} \text{=} V_{\text{DD}} - \frac{\mu C_{\text{OX}} W}{2L} \left[ V_{\text{INQ}} \text{-} V_{\text{SS}} \text{-} V_{\text{T}} \right]^2 R \end{split}$$

Near the Q-point, small signals have linear relationship:

$$V_{\text{outsmall}} = V_{\text{outq}} - V_{\text{outq}} \cong A_{\text{v}} \cdot (V_{\text{IN}} - V_{\text{INQ}}) = A_{\text{v}} V_{\text{m}} \sin \omega t$$

$$V_{\text{outsmall}} \cong A_{\text{v}} V_{\text{INsmall}}$$

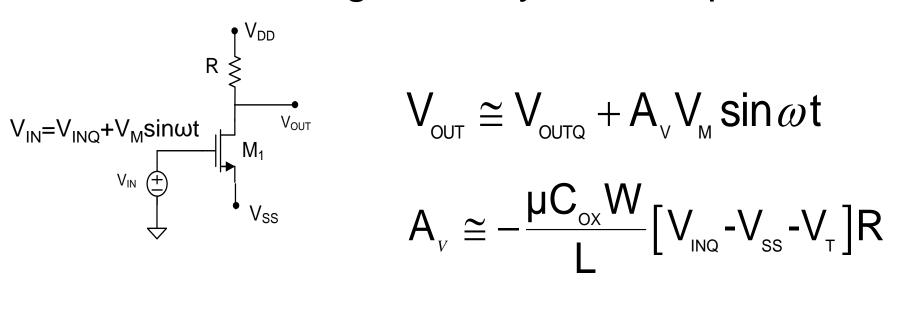
$$A_{v} \cong -\frac{\mu C_{ox}W}{L} [V_{inQ} - V_{ss} - V_{T}]R$$



But – this expression gives little insight into how large the gain is!

Can the gain be made arbitrarily large by simply making R large?

Observe increasing R with W,L, and V<sub>SS</sub> fixed will change Q-point Difficult to answer this question with the information provided!



$$V_{\text{out}} \cong V_{\text{outq}} + A_{\text{v}} V_{\text{M}} \sin \omega t$$

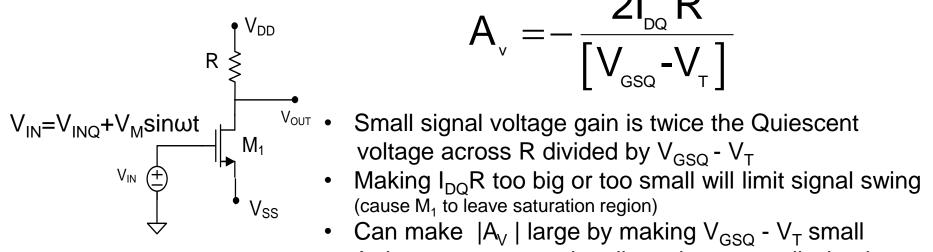
$$A_{v} \cong -\frac{\mu C_{ox}W}{L} [V_{inq} - V_{ss} - V_{T}] R$$

$$I_{DQ} = \frac{\mu C_{OX} W}{2L} (V_{INQ} - V_{SS} - V_{T})^{2}$$

But recall:

Thus, substituting from the expression for  $I_{DQ}$  we obtain

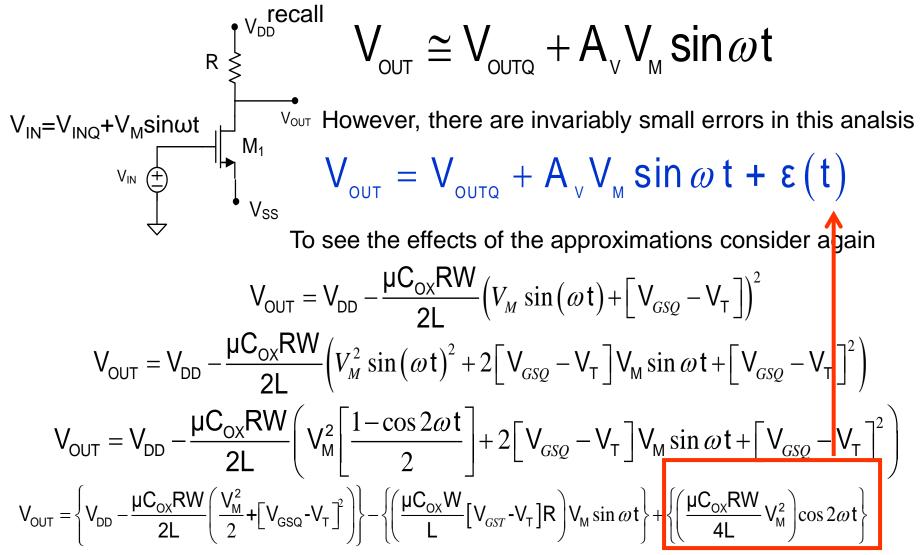
$$A_{v} = -\frac{2I_{DQ}R}{\left[V_{INQ}-V_{SS}-V_{T}\right]} = -\frac{2I_{DQ}R}{\left[V_{GSQ}-V_{T}\right]}$$



$$A_{v} = -\frac{2I_{DQ}R}{\left[V_{GSQ}-V_{T}\right]}$$

- Can make  $|A_V|$  large by making  $V_{GSQ}$   $V_T$  small
- A<sub>V</sub> increases proportionally to the power dissipation (from supply) for fixed V<sub>GSO</sub>
- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

(Consider what was neglected in the previous analysis)



Note presence of second harmonic distortion term!

Nonlinear distortion term 
$$V_{\text{IN}} = V_{\text{INQ}} + V_{\text{M}} \sin \omega t$$

$$V_{\text{IN}} = V_{\text{INQ}} + V_{\text{M}} \sin \omega t$$

$$V_{\text{OUT}} = V_{\text{OUTQ}} + A_{\text{V}} V_{\text{M}} \sin \omega t + \epsilon (t)$$

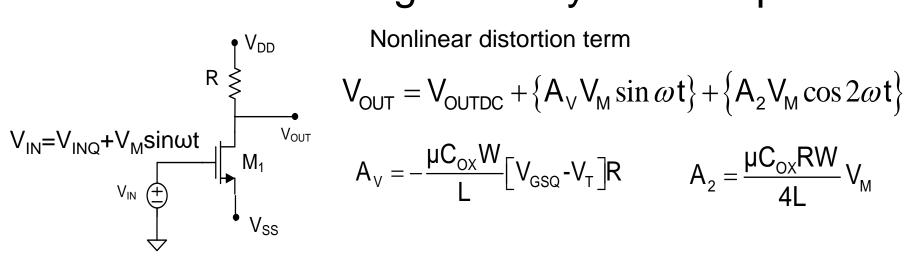
$$V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{OX}} RW}{2L} \left( \frac{V_{\text{M}}^2}{2} + \left[ V_{\text{GSQ}} \cdot V_{\text{T}} \right]^2 \right) \right\} - \left\{ \left( \frac{\mu C_{\text{OX}} W}{L} \left[ V_{\text{GSQ}} \cdot V_{\text{T}} \right] R \right) V_{\text{M}} \sin \omega t \right\} + \left\{ \left( \frac{\mu C_{\text{OX}} RW}{4L} V_{\text{M}}^2 \right) \cos 2\omega t \right\}$$

$$V_{\text{OUTDC}} = V_{\text{OUTQ}} - \frac{\mu C_{\text{OX}} RW}{4L} V_{\text{M}}^2$$

$$A_{\text{V}} = -\frac{\mu C_{\text{OX}} W}{L} \left[ V_{\text{GSQ}} \cdot V_{\text{T}} \right] R$$

$$A_{\text{2}} = \frac{\mu C_{\text{OX}} RW}{4L} V_{\text{M}}$$

$$V_{\text{OUT}} = V_{\text{OUTDC}} + \left\{ A_{\text{V}} V_{\text{M}} \sin \omega t \right\} + \left\{ A_{\text{2}} V_{\text{M}} \cos 2\omega t \right\}$$



$$V_{OUT} = V_{OUTDC} + \{A_{V}V_{M}\sin\omega t\} + \{A_{2}V_{M}\cos2\omega t\}$$

$$A_{V} = -\frac{\mu C_{OX}W}{L} \left[ V_{GSQ} - V_{T} \right] R \qquad A_{2} = \frac{\mu C_{OX}RW}{4L} V_{M}$$

**Total Harmonic Distortion:** 

Recall, if 
$$x(t) = \sum_{k=0}^{\infty} b_k \sin(k\omega T + \phi_k)$$
 then 
$$THD = \frac{\sqrt{\sum_{k=2}^{\infty}} b_k^2}{|b_1|}$$

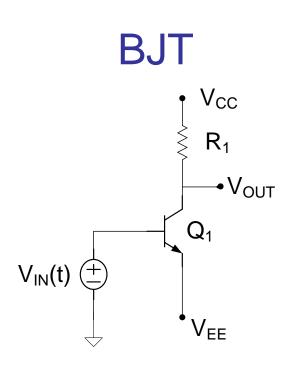
Thus, for this amplifier, as long as M₁ stays in the saturation region

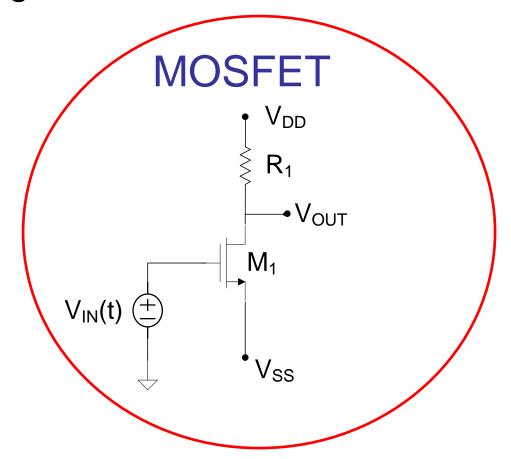
THD = 
$$\frac{\sqrt{(A_2 V_M)^2}}{|A_V V_M|} = \frac{A_2}{|A_V|} = \frac{\frac{\mu C_{OX} W}{4L} R V_M}{\frac{\mu C_{OX} W}{L} R (V_{GSQ} - V_T)} = \frac{V_M}{4(V_{GSQ} - V_T)}$$

Distortion will be small for  $V_M << (V_{GSO} - V_T)$ 

Distortion will be much worse (larger and more harmonic terms) if M<sub>1</sub> leaves saturation region.

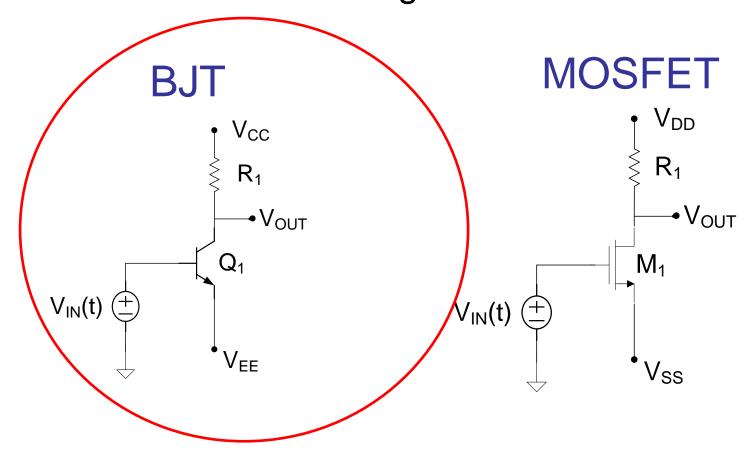
#### Consider the following MOSFET and BJT Circuits



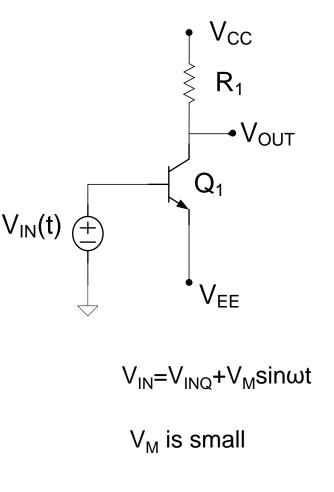


- Analysis was very time consuming
- Issue of operation of circuit was obscured in the details of the analysis

#### Consider the following MOSFET and BJT Circuits

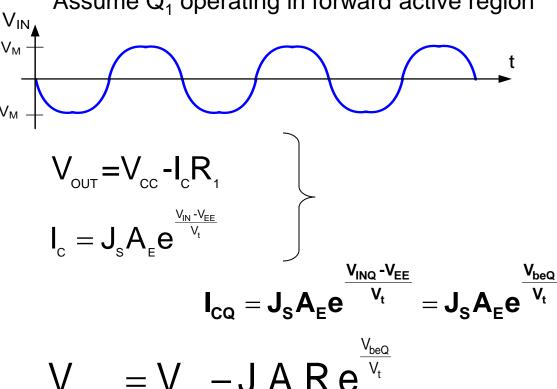


### Small signal analysis using nonlinear models



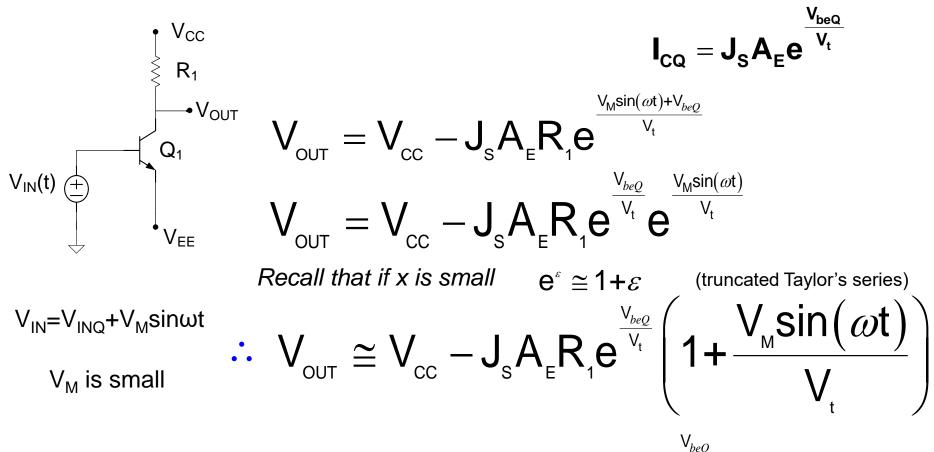
By selecting appropriate value of  $V_{SS}$ ,  $M_1$ will operate in the forward active region

Assume Q<sub>1</sub> operating in forward active region



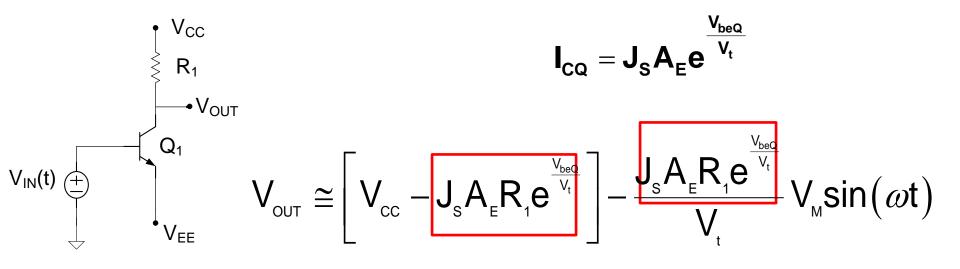
$$egin{aligned} V_{ ext{outq}} &= V_{ ext{cc}} - J_{ ext{s}} A_{ ext{E}} R_{ ext{1}} e^{rac{V_{ ext{beQ}}}{V_{ ext{t}}}} \ V_{ ext{out}} &= V_{ ext{cc}} - J_{ ext{s}} A_{ ext{E}} R_{ ext{1}} e^{rac{V_{ ext{msin}(\omega t) + V_{ ext{beq}}}{V_{ ext{t}}}} \end{aligned}$$

### Small signal analysis using nonlinear models



$$V_{\text{out}} \cong \left[ V_{\text{cc}} - J_{\text{s}} A_{\text{E}} R_{\text{1}} e^{\frac{V_{\text{beQ}}}{V_{\text{t}}}} \right] - \frac{J_{\text{s}} A_{\text{E}} R_{\text{1}} e^{\frac{V_{\text{beQ}}}{V_{\text{t}}}}}{V_{\text{t}}} V_{\text{m}} \sin(\omega t)$$

## Small signal analysis using nonlinear models



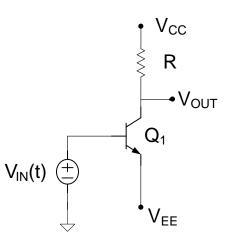
$$V_{IN}=V_{INQ}+V_{M}sin\omega t$$

V<sub>M</sub> is small

$$V_{\text{OUT}} \cong \begin{bmatrix} V_{\text{CC}} - I_{\text{CQ}} R_1 \\ Quiescent Output \end{bmatrix} - \begin{pmatrix} I_{\text{CQ}} R_1 \\ V_t \end{pmatrix} V_{\text{M}} \sin(\omega t)$$
ss Voltage Gain

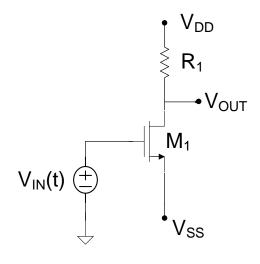
#### Comparison of Gains for MOSFET and BJT Circuits

#### BJT



$$A_{_{VB}} = -\frac{I_{_{CQ}}R}{V_{_{t}}}$$

#### **MOSFET**



$$\mathsf{A}_{\scriptscriptstyle \mathsf{VM}} = -rac{2\mathsf{I}_{\scriptscriptstyle \mathsf{DQ}}\,\mathsf{R}_{\scriptscriptstyle \mathsf{I}}}{V_{\scriptscriptstyle \mathit{GSQ}} - V_{\scriptscriptstyle \mathit{T}}}$$

If  $I_{DQ}R_1 = I_{CQ}R = 2V$ ,  $V_{GSQ} - V_T = 1V$ ,  $V_t = 25mV$ 

$$A_{VB} = -\frac{I_{CQ}R}{V_{T}} = -\frac{2V}{25mV} = -80 \qquad A_{VM} = -\frac{2I_{DQ}R_{T}}{V_{GSQ} - V_{T}} = -\frac{4V}{1V} = -4$$

$$A_{VM} = -\frac{2I_{DQ}R_{1}}{V_{GSO}-V_{T}} = -\frac{4V}{1V} = -4$$

Observe A<sub>VB</sub>>>A<sub>VM</sub>

Due to exponential-law rather than square-law model

# Operation with Small-Signal Inputs

- Analysis procedure for these simple circuits was very tedious
- This approach will be unmanageable for even modestly more complicated circuits
- Faster analysis method is needed!



Stay Safe and Stay Healthy!

# End of Lecture 23